SELÇUK UNIVERSITY

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CONSTRUCTIVE MATHEMATICAL ANALYSIS

2nd INTERNATIONAL WORKSHOP: CONSTRUCTIVE MATHEMATICAL ANALYSIS

06-08 July 2023 Konya-TÜRKİYE

ABSTRACT BOOK

The Second International Workshop: Constructive Mathematical Analysis

Abstract Book

July 6-8, 2023
Konya, Türkiye

CMA
CONSTRUCTIVE MATHEMATICAL ANALYSIS

FOREWORDS

Dear Participants,

First of all, we would like to thank you for coming today to participate at this opening ceremony and we wish to welcome you to Türkiye and Konya. We hope we will have a good time during this workshop.

The purpose of this workshop is to bring together experts and young analysts from all over the world working in different fields of mathematics and its applications to present their researches, exchange new ideas, discuss challenging issues, foster future collaborations and interact with each other.

This workshop allows the participation of a number prominent experts from different countries who will present works on different fields of mathematics, especially approximation theory, fixed point theory, nonlinear analysis, optimization, summability theory, sequence spaces, dynamical systems and their applications, and also algebra, geometry.

We also thank invited speaker distinguished **Prof. Ali Aral**, distinguished **Prof. Harun Karslı**, distinguished **Prof. Carlo Bardaro**, distinguished **Prof. Gianluca Vinti**, distinguished **Prof. Calogero Vetro** and distinguished **Prof. Danilo Costarelli** for contribution to the our workshop.

The papers presented in this workshop will be considered in the journal *Constructive Mathematical Analysis*.

This booklet contains the titles and abstracts of almost all invited and contributed talks at the The Second International Workshop: Constructive Mathematical Analysis and will be made available on the workshop website. All talks will take place in Selçuk University, Faculty of Science, Department of Mathematics, Ali Rıza Çetik Workshop Hall, Alaaddin Keykubat Campus, 42003, Selçuklu/Konya, Türkiye.

We wish everyone a fruitful workshop and pleasant memories in Konya, Türkiye.

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Contents

On a New Approach in the Space of Measurable Functions

Ali Aral

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key-words: Modulus of continuity, Mellin convolution type integral global smoothness.

Abstract:
In this speech, we present a new modulus of continuity for locally integrable function spaces which is effected by the natural structure of the L_p space. After basic properties of it are expressed, we provide a quantitative type theorem for the rate of convergence of Mellin convolution type integral operators and iterates of them. Moreover, we state their global smoothness preservation property including the new modulus of continuity. Finally, the obtained results are performed to the Mellin Gauss-Weierstrass operators .

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Exponential Sampling with a Multiplier

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key-words: Polar-analytic functions, Mellin–Paley-Wiener spaces, Mellin-Bernstein spaces, multipliers, exponential sampling.

Abstract:
The exponential sampling formula, in classical, approximate and generalized forms has been deeply studied in the last years, especially for what concerns the interconnections between them. Here we will describe some interesting links between the classical and the generalized exponential sampling formula. The basic motivation for this kind of study, relies on the fact that the exponential sampling formula, in its classical form, has some limitations. By incorporating a Mellin bandlimited multiplier, we extend it to a wider class of functions with a series that converges faster. This series is a generalized exponential sampling series with some interesting properties. Moreover, under a side condition, any generalized exponential sampling series that is interpolating can be generated by a Mellin bandlimited multiplier. For an error analysis, we consider a truncated series with $2N + 1$ terms and look for a highest speed of convergence as $N \to \infty$. We show by using a certain non-bandlimited multiplier, which introduces in addition an aliasing error, that we can achieve a higher rate of convergence to the function, namely $\mathcal{O}(e^{-\alpha N})$ with $\alpha > 0$, than with the truncated series of an exact formula.

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Estimates of the Approximation Error for Families of Neural Network Operators

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Abstract:
In this talk we present quantitative estimates for the error of approximation with respect to the sup-norm, as well as, to the L^p-norm, achieved by neural network (NN) operators activated by sigmoidal functions. The above estimates have been given by means of suitable moduli of smoothness of the approximated functions. Particular emphasis will be given to the very delicate case of functions belonging to L^p-spaces, $1 \leq p < +\infty$.

A Mathematical Model for the Effects of Wavelets and the Analysis of Neural Network Operators Described Using Wavelets

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key-words: Wavelet, neural network operators, approximation.

Abstract:
It is very well-known that wavelets have the great advantage of being able to separate and identify fine details in a signal or a function. One of the main advantages of wavelets compared to the Fourier analysis and its related theories is that they offer simultaneous localization in the time and frequency domain. The second main advantage of wavelets is that they are computationally very fast and detailed when using wavelet expansions and transformations. In the present study we will deal with the linear approximation operators constructed by compactly supported Daubechies wavelets. In details, we will reconstruct neural network operators, where location and time are very important and effective, with the help of wavelets, and we will examine and analyse various properties of the wavelet type extension of the neural network operators.

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Nonlinear Dirichlet Problems with Dependence on the Gradient

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Abstract:
We discuss nonlinear systems of elliptic equations under a Dirichlet boundary condition. The principal differential operators are based on the p-Laplacian, and the reaction terms exhibit dependence on both the solution and its gradient. The subsolution-supersolution method permits us to prove both the existence and location of weak solutions. The strategy of proof relies on suitable truncation and cut-off functions, together with Nemytskij operators appropriately involved in working with certain auxiliary problems.

Mathematical Models Based on the Approximation of Discrete Operators for the Processing of Thermographic Images

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Abstract:
During this talk, I will discuss some mathematical models for the reconstruction and the processing of digital images, focusing in particular on one recently introduced, based on discrete operators of sampling type and successfully applied in various fields. I will examine it by dealing in particular with the processing of thermographic imagesfor the study of the seismic vulnerability of buildings.

Fixed Point Results for Orthogonal Metric Spaces

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key-words: Fixed point, orthogonal metric space, O-complete metric space.

Abstract:
Metrical fixed point theory is started by Banach in 1922 [3]. Banach's Contraction Principle says that, whenever (X, d) is complete, then any contraction selfmap of X has a unique fixed point. Many authors obtained some fixed point theorems for several classes of contractive mappings, see [1, 2]. Lately, for the first time, Gordji et. al. [1] extended the literature on metric space by introducing the concept of orthogonality and obtained the fixed point result. The aim of this talk is to obtain some fixed point results on orthogonal complete metric space and give some corollaries which are well-known results in the literature. Finally, we give an illustrative example.

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Recent Results on Weighted Approximation by Sampling Type Series

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key-words: Sampling operators, weighted spaces, ımage processing

Abstract:
This talk details our recent research on weighted approximation through the lens of sampling type series, with an emphasis on the utilization of generalized sampling operators. We offer a comprehensive solution approach to several issues such as the approximation properties of sampling operators and their modifications in weighted spaces. The discourse covers the determination of the convergence rate, and the calculation of the upper-lower error magnitude of approximation, facilitated by appropriate continuity modules and functionals. Simultaneous approximations for sampling series are also proposed, focusing on the context of weighted spaces and the approximation of sampling Kantorovich operators within these spaces. The concluding part of our work introduces the construction of multivariate sampling series and showcases their applicability in numerous disciplines.

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Some Properties of Squared Chlodovsky Operators

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key-words: Linear positive operators, Chlodovsky operators, rate of convergence.

Abstract:
For a function f defined on $[0,\infty)$ and bounded on every finite interval $[0,b]\subset[0,\infty)$, the classical Chlodovsky operators [1] are defined by

$$(C_n f)(x)=\sum_{k=0}^{n} f\left(\frac{b_n}{n}k\right) p_{k,n}(\frac{x}{b_n}),$$

where $p_{k,n}(x)=\left(\begin{array}{c} n \\ k \end{array}\right) x^k(1-x)^{n-k}$, $0\leq x\leq 1$, and $(b_n)_{n=1}^{\infty}$ is a positive increasing sequence of reals with the properties

$$\lim_{n\to\infty} b_n=\infty\ ,\ \lim_{n\to\infty}\frac{b_n}{n}=0.$$

The main objective of this study is to define a new sequence of positive linear operators by means of the squared Chlodovsky basis functions. And we give some approximation properties of these operators.

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Time-Ordered Hyperbolic Evolution Systems

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key-words: Time-ordered operator, Feynman-Dyson space, hyperbolic evolution system.

Abstract:
In this study, in order to develop a theory for the homogeneous and inhomogeneous hyperbolic initial value problems, respectively, on the Feynman-Dyson space, time-ordered evolutions were created using semi-groups of operators, and their stability and asymptotic properties were investigated.

Acknowledgements:
"This work was supported by Research Fund of the Yildiz Technical University. Project Number:FBA-2023-5765".

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On New Means Generated by Inverse of Eigenfunctions of (p, q)-Laplacian

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key-words: Hypergeometric functions, generalized trigonometric functions, eigenfunctions.

Abstract:
Motivated by the work of [1, 3], here author generalizes logarithmic mean L, Neuman-Sándor M, two Seiffert means P and M as an application of generalized trigonometric and hyperbolic functions with two parameters. Moreover, several two-sided inequalities involving these generalized means are established.

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Approximation Properties of Classical and Weak Derivatives of Sampling Kantorovich Operators

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key-words: Sampling Kantorovich operators, approximation of signals, Sobolev spaces

Abstract:
In this talk, we will analyze some approximation results of classical and weak derivatives of Kantorovich-type sampling operators. Specifically, we will explore how the n-th derivative of the operator can pointwise reconstruct the n-th derivative of a function at every point where such derivative exists. Additionally, we will examine the behavior of these operators in the presence of a function with not differentiable points. Lastly, we will present convergence results for functions belonging to integer and fractional Sobolev spaces.

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Approximation by Kantorovich Type Cheney-Sharma Chlodowsky Operators

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key-words: Kantorovich operators, Cheney Sharma operators, weighted Korovkin theorem, modulus of continuity.

Abstract:
The main purpose of this paper is to define Kantorovich type Cheney-Sharma Chlodowsky operators. In this paper, we analyze approximation properties of these new operators with the help of weighted Korovkin type theorem. Morever, we also give the rate of convergence by means of the modulus of continuity.

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A Generalization of the Szász-Baskakov Operators

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key-words: Szász-Baskakov operators, weighted approximation, Voronovskaya-type theorem.

Abstract:
In this talk, we provide a generalization of the Szász-Baskakov operators. We look into the convergence rate and approximation properties in weighted space. We use a few polynomials to illustrate theoretical findings. Finally, using various error estimates tables, we demonstrate the convergence of the suggested operators to some specific functions.

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Some Fixed Point Results on Weak Partial Metric Space

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key-words: Fixed point, partial metric space, weak partial metric space.

Abstract:
In 1994, S. G. Matthews [6] introduced the concept of partial metric space. In a partial metric spaces, the distance of a point in the self may not be zero. After the definition of partial metric space, Matthews proved a partial metric version of Banach's fixed point theorem. In [1, 5, 7] have been made some generalizations of the result of Matthews. Heckmann [4] introduced the concept of weak partial metric space, which is a generalized version of Matthews' partial metric space by omitting the small self-distance axiom. It is clear that every partial metric space is a weak partial metric space, but the converse may not be true. Some results for mappings in weak partial metric spaces have been obtained by [2] and [3]. In this talk, Our purpose in this talk is to examine new fixed point problems in weak partial metric spaces and to provide a supporting example.

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On Generalized Deferred Statistical Boundedness of Order α

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key-words: Difference sequence, deferred statistical convergence, deferred statistically cauchy.

Abstract:
In this paper, using the generalized difference operator Δ^m, we introduce the concept of Δ^m-deferred statistical boundedness of order α and give some inclusion relations between Δ^m-deferred statistical convergence of order α and Δ^m-deferred statistical boundedness of order α.

Acknowledgement: This work was partially supported by the Management Union of the Scientific Research Projects of Firat University under the Project Number: FUBAB FF.22.29. We would like to thank Firat University Scientific Research Projects Unit for their support.

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Almost a Hilbert Space

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key-words: Duality, M-basis, Schatten class.

Abstract:
Kuelbs [1] proved that every separable Banach space $\mathcal{B}$ can be densely and continuously embedded in a Hilbert space $\mathcal{H}$. If $\mathcal{L}[\mathcal{B}]$ is the bounded linear operators on $\mathcal{B}$, in this talk I will we prove following:

1. For each $u \in \mathcal{B}$, there exists a constant c_u and a semi-inner product $[\,\cdot\,, u]_z$ generated by a bounded linear functional $u_z^* \in \mathcal{B}^*$ such that $u_z^*(v) = [v, u]_z = c_u(v, u)_{\mathcal{H}}$ for every $v \in \mathcal{B}$ (see Lumer [3]).
2. $\mathcal{L}[\mathcal{B}] \subset \mathcal{L}[\mathcal{H}]$ as a continuous dense embedding and for each $A \in \mathcal{L}[\mathcal{B}]$ there exists $A^* \in \mathcal{L}[\mathcal{B}]$ with $(A^*A)^* = A^*A$ (see Lax [2]).
3. Every compact operator on $\mathcal{B}$ is the limit of a sequence of operators of finite rank without assuming that $\mathcal{B}$ has a basis.
4. The Schatten class of operators $\mathbb{S}_p[\mathcal{H}] \supset \mathbb{S}_p[\mathcal{B}]$ as a continuous dense embedding and $\bar{A} \in \mathbb{S}_p[\mathcal{H}]$ if and only if it's restriction $A \in \mathbb{S}_p[\mathcal{B}]$, with $\|\bar{A}\|_p^{\mathcal{H}} = \|A\|_p^{\mathcal{B}}$, for each $p \in [0, \infty]$.

These results lead us to conclude that every separable Banach space is almost a Hilbert space. (If time permits, I will use $\mathcal{C}[0, 1] \subset L_2[0, 1]$ to provide concrete examples.)

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Pointwise Convergence of α-Bernstein-Durrmeyer Operators with respect to Arbitrary Measures

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key-words: Pointwise convergence, α-Bernstein-Durrmeyer operators, Positive linear operators.

Abstract:
The α-Bernstein-Durrmeyer operators $\mathfrak{D}^{\alpha}_{d,\mathbf{Z}}$ corresponding to a collection $\mathbf{Z}$ of arbitrary measures are generalized in this work. This study discusses the pointwise convergence of $\mathfrak{D}^{\alpha}_{d,\mathbf{Z}}$ and provides convergence criteria on the collection of measures for the same. A more general and broader collection is introduced as compared to the earlier available literature. The generalization generates a variety of Bernstein-like operators with its α variation for particular values of the measures taken into account.

Comparative Analysis of Nonlinear SEIRD and SVEIRD Models to Understand the Dynamics of Covid-19 Spread in Alberta

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key-words: SEIRD and SVEIRD models, stability, COVID-19.

Abstract:
The dynamics of infectious diseases can be effectively investigated with the aid of mathematical models, providing useful information to help control disease spread. This study aims to characterize the dynamic nature of Covid-19 in Alberta by comparing the existing nonlinear Susceptible-Exposed-Infected-Recovering-Dead (SEIRD) model against the innovative nonlinear SVEIRD model, which incorporates a vaccination compartment. The generation operator method is applied to determine the basic reproduction number of this new model. A comprehensive analytical examination of all potential steady states within the dynamic SVEIRD model is conducted. Stability conditions are derived utilizing the Routh–Hurwitz stability theories. Numerical simulations for both dynamic models are provided, relying on the data from the reported Covid-19 cases in Alberta. These findings help underscore the theoretical implications drawn from the model.

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Some Applications of Semi-Exponential Post-Widder Operators

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key-words: Semi-exponetial Post-Widder operators, difference, weighted convergence.

Abstract:
In 2005 Tyliba and Wachnicki introduced an extension of Szász-Mirakyan and Weierstrass operators. Later, the modification of Post-Widder operators arised in the paper [3] as follows

$$(P_\lambda^\beta f)(x) = \int_0^\infty k_\lambda^\beta(x,t) f(t) dt,$$

where the kernel

$$k_\lambda^\beta(x,t) = \frac{\lambda^\lambda}{x^\lambda e^{\beta x}} \sum_{k=0}^{\infty} \frac{(\lambda\beta)^k}{k!\Gamma(\lambda+k)} e^{-\lambda t/x} t^{\lambda+k-1},$$

satisfies the partial differential equation:

$$\frac{\partial}{\partial x} k_\lambda^\beta(x,t) = \left[\frac{\lambda(t-x)}{x^2} - \beta\right] k_\lambda^\beta(x,t),$$

for $\beta, \lambda > 0$, $x \in I := [0, +\infty)$ and $f \in C(I)$.

From then on, we call the new version, the semi-exponential Post-Widder operators because, for $\beta = 0$, we get exponential operators introduced by Ismail and May in 1978. In 2022 Abel et al. [1] and Gupta and Milovanović [2] invented all remaining semi-exponential operators from available exponential ones.

In the talk we shall present the difference between the two operators. We will point out that in the case of the Post-Widder operators, the difference with other operators is not evident due to the purely integral term of the operators. However, considering the semi-exponential version of the Post-Wdder operators, one can obtain the difference with other operators.

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On The Second Regularized Trace Formula of A Second Order Differential Operator

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key-words: Spectrum, resolvent, regularized trace.

Abstract:
In this work we find a formula for second regularized trace of a Sturm-Liouville differential operator with bounded operator coefficient which is formed by differential expression

$$D(y) = -y''(t) + Q(t)y(t)$$

and with the periodic boundary conditions

$$y(0) = y(\pi); \quad y'(0) = y'(\pi)$$

in the seperable Hilbert space $H_1 = L_2(H; [0, \pi])$.

Acknowledgements:
This work has been supported by Research Fund of the Yildiz Technical University. Project Number: FBA-2023-5415

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Shape Control of Novel Generalized Trigonometric Bézier Curves and their Applications

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key-words: Generalized Bézier curves, trigonometric Bézier curves, parametric curves, shape parameters, curve modeling.

Abstract:
In this study, we introduce new basis functions to construct generalized trigonometric (GT-Bézier) Bézier curves using shape parameters that assist in determining the ideal curves when tackling designing and modeling tasks. The features acquired by novel GT-Bézier curves are investigated parallel to classical Bézier curves. To create intricate structures, multiple GT-Bézier curves are combined by using parametric continuities. Graphical examples are provided along with an analysis of the constraints required to attain these continuities. It is observed that the novel GT-Bézier curves give better approximation as compared to classical Bézier curves and are closer to the control polygon for the higher values of the shape parameters. Several formations are addressed in relation to applications of the suggested GT-Bézier curves in graphing shapes such as free-form structures and font designing. For some values of the shape parameters, the proposed basis appears to be more effective than the earlier known bases.

Comparison of Imputation and Weighting Methods in Estimation of a Finite Population Mean under Random Non-response

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key-words: Sampling with replacement, two-stage cluster sampling, bias and mean squared error.

Abstract:
Developing estimators of finite population parameters such as mean, variance and asymptotic mean squared error has been one of the core objectives of sample survey theory and practice. Sample survey practitioners need to assess the properties of these estimators so that better ones can be adopted. In survey sampling, the occurrence of non-response affects inference and optimality of the estimators of finite population parameters. It introduces bias and may cause samples not to follow the the distributions determined by the original sampling design. To compensate for random non-response, imputation methods and weighting techniques can be used. In this paper, a comparison between these two methods of compensating for non-response has been done in two-stage cluster sampling. Simulation results reveal tighter confidence interval lengths, smaller mean squared error values for the estimators developed under the weighting method than its rival estimators obtained using imputation method. Under mild assumptions, the weighting method is shown to be more efficient than the imputation techniques in estimating a finite population mean.

On Fractional-type Exponential Sampling Kantorovich Series

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key-words: Exponential sampling Kantorovich series, fractional-type integral operators, modulus of continuity, logarithmic weighted space of functions.

Abstract:
In this talk, we present the construction of a new family of exponential sampling Kantorovich operators based on suitable fractional-type integral operators and we give main approximation results of the new operators such as pointwise/uniform convergence and then by using certain modulus of continuity, rate of convergence. Moreover, we deal with the logarithmic weighted space of continuous functions.

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Kantorovich-Stancu Type Lototsky-Chlodowsky Operators

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key-words: Bernstein-Chlodowsky polynomials, Lototsky-Chlodowsky Operators, Kantorovich-Stancu Type Operators, Korovkin type approximation theorem.

Abstract:
In this study, we define Kantorovich-Stancu Type Lototsky-Chlodowsky operators and obtain some approximation properties of these operators.

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Design of a Statistical Test Suite. Case of Study

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key-words: Correlation, Hypothesis testing, Mutual Information, Test battery, Test suite.

Abstract:
In several areas of knowledge it is necessary to work with random sequences (or at least that behave as such). The main tools for the verification of the fundamental properties that these sequences must verify (randomness, uniformity) are hypothesis tests. These tests are usually grouped in sets called suites or batteries. One of the current lines of research focuses on the analysis and design of such suites. In this paper we will analyse the statistical techniques most commonly used in this analysis and we will show a case study of interdependence analysis in a suite.

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On the Uniform Summation of Jacobi Series

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key-words: Summation, Jacobi-Fourier series, Cesaro, matrix transformation.

Abstract:
The paper deals with the study of a matrix method of summation which is stronger than all the Cesaro's methods and assures the uniform summation of the to Jacobi-Fourier series attached to continuous functions.

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A Note on Generation of All Pythagorean Triples

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key-words: Primitive Pythagorean triples; non-primitive Pythagorean triples; multipliers, transpositions, qryptography

Abstract:
There exist several techniques used to generate Pythagorean triples. The most effective formula for generating Pythagorean triples is the Euclid's formula. Whereas the Euclid's formula generate infinitely many Pythagorean triples, it does not generate all of them. For instance, the Euclid's formula generates the triple $(3,\ 4,\ 5)$ but does not generate $(4,\ 3,\ 5)$, in which case a transposition is needed. In addition, the triple $(9,\ 12,\ 15)$ cannot be generated directly from the Euclid's formula but rather a multiplier to the triple $(3,\ 4,\ 5)$ does so. In this note, we contribute to the old problem of generating all Pythagorean Triples, primitive and non-primitive, without using a transformation and without using a multiplier.

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A Result for Single and Multi-Valued Mapping on Orthogonal Metric Space

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key-words: Fixed point, $F-$contraction, orthogonal metric space, orthogonal $F-$contraction.

Abstract:
Fixed point theory is parts of the crucial and traditional theories in mathematics and have a broad set of applications. The beginning of the metric fixed point theory is the famous Banach Contraction Principle given by Banach [1] in 1922. In this theory, contraction is one of the main means of proving the existence and uniqueness of a fixed point. Subsequently, the crucial role of the principle in existence and uniqueness problems arising in mathematics has been realized which fact directed the researchers to extend and generalize the principle in many ways. In 2017, Gordji et al. [2] defined the concept of an orthogonal set and gave an extension of the Banach Contraction Principle in orthogonal metric spaces and also they gave the applications. On the other hand, Sawangsup et al. [3] gave a generalization $F-$contraction mapping in orthogonal metric spaces in 2020. In this talk, we consider rational type $F-$contraction on orthogonal metric space and establish the existence of fixed point of such mapping and finally, we give an supporting example.

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On a New Approach of the Mellin Transforms

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key-words: Mellin convolution, modulus of continuity, order of approximation.

Abstract:
In this presentation, we express a new approach of Mellin transforms. Later, the weighted logarithmic modulus of continuity is given. Furthermore, we state a Voronovskaya type theorem. Finally, we indicate that the obtained results are applied to the Mellin-Gauss-Weierstrass operator.

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Analysis with the Group $ax + b$ and with the Mellin Harmonic Oscillator

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key-words: Representations of the group $ax + b$ Sobolev, Besov, and Paley-Wiener spaces, K-functor, modulus of continuity, Mellin harmonic oscillator, Paley-Wiener functions, Jackson-type inequality, frames.

Abstract:
The talk introduces Sobolev and Besov spaces associated with a representation of the Lie group $ax + b$ in a space of functions defined on the half-line. The Besov spaces are described in terms of the K-functional, in terms of a relevant moduli of continuity, and in terms of a frequency-localized Hilbert frames. It is observed that a Laplace operator associated with this representation can be treated as a Mellin harmonic oscillator. Using this operator we introduce relevant Paley-Wiener spaces and develop an approximation theory in which our Besov spaces appear as approximation spaces. In particular, a direct approximation theorem is proved in the form of a Jackson-type inequality.

Stancu Type Dunkl Generalization of Szasz Durrmeyer Operators Involving Two-Variable Hermite Polynomials

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key-words: Dunkl exponential, Szasz operators, modulus of continuity, Lipschitz functions, two-variable hermite polnomials, Stancu type Dunkl generalization of Szasz-Durrmeyer operators.

Abstract:
The aim of the present paper is to introduce a Stancu type Dunkl generalization of Szasz-Durrmeyer operators including two-variable Hermite polynomials defined by Krech [6]. Then, we give approximation properties for these operators with the help of Korovkin theorem .Furthermore, we introduce other approximation results via the class of Lipschitz functions, classical modulus of continuity, second modulus of continuity and Peetre's K-functional.

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Asymptotic Analysis of Non-local Boundary-Value-Problems with Eigenvalue Parameter Contained in the Boundary Conditions

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key-words: Sturm–Liouville problems, non-local boundary conditions, asymptotics of eigenvalues and eigenfunctions.

Abstract:
Sturm–Liouville problems with eigenvalue-parameter-dependent boundary conditions (BCs) arise upon separation of variables in the one- dimensional wave and heat equations and have many applications including vibration problems involving various types of loads, diffusion of water vapour through a porous membrane and several electric circuit problems involving long cables. Asymptotic formulas of eigenvalues and eigenfunctions of such kind of problems with local type BCs obtained in [1]. Asymptotic formulas of eigenvalues and eigenfunctions of problems with non-local type boundary conditions (NBCs) which includes eigenvalue parameter only in the differential equation obtained in [2]. In this study we shall investigate non-local eigenvalue problems which consist of Sturm–Liouville equation

$$-u''(t)+q(t)u(t)=\lambda^2 u(t)$$

on $[0,1]$ with one classical eigenvalue-parameter-dependent BC and another eigenvalue-parameter-dependent NBC

$$u'(0)-h\lambda u(0)=0, \quad u'(1)+(H_1\lambda+H_2)u(\xi)=0, \quad \xi\in(0,1)$$

where the real-valued function $q\in C[0,1]$, $\lambda=x+\imath y$ is a complex spectral parameter; $x,y,h,H_1,H_2\in\mathbb{R}$.

As a main result, we obtain some asymptotic formulas of eigenvalues and eigenfunctions for this problem.

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Loewner Theory and Free Boundary Problems

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key-words: Loewner Theory, univalent functions, free boundary problem.

Abstract:
In [1] Beurling gives a powerful existence theorem for some free boundary problems in the complex plane. We apply Loewner Thoeory to give a new uniqueness theorem for the solutions of these problem found using his method. We then show some applications of this method.

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Convergence of Mao Operators for Functions with Derivatives of Bounded Variation

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key-words: Approximation, bounded variation, Mao Operators, rate of convergence, total variation.

Abstract:
This article delves into the investigation of the approximation properties pertaining to the class of Mao operators. Specifically, we aim to evaluate the rate of pointwise convergence exhibited by these Mao operators, denoted as $M_{n,k}$, when applied to functions with derivatives of bounded variation.

Sampling Type Series: Approximation via Riemann-Liouville Fractional Integral

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key-words: Fractional integrals, weighted approximation, sampling type series.

Abstract:
In this paper, we present sampling type series based on Riemann-Liouville fractional integral. Our study focuses on investigating the approximation properties of this series. The second part of the paper delves into the weighted approximation of the family of operators. At the end, we give specific examples of kernels which satisfies the obtained results with numerical tables and graphical representations.

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Some Properties of Modified Bernstein Operators

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key-words: Bernstein operators, modified Bernstein operators, eigenvalues, eigenvectors.

Abstract:
The Bernstein operators play a fundamental role in the theory of linear positive operators. These polynomials have considerable interest among researchers, leading to deep investigations from different perspectives, and motivating them to introduce various generalizations and analogues. The first modification was introduced in 1932 by Bernstein himself [1] aiming to speed up the rate of approximation for smooth functions. This modification is given by

$$B_{n,1}(f;x) = B_{n,2}(f;x) = B_n(f;x),$$

$$B_{n,m+1}(f;x) = B_n(f;x) - \sum_{k=2}^{m} \frac{1}{k!} B_n((t-x)^k;x) B_{n,m+1-k}(f^{(k)};x), \quad m \geqslant 2.$$

In recent decades, based on the work of Cooper and Waldron, numerous new researches have emerged to investigate the eigenstructure of the Bernstein type operators. The purpose of this talk is to present the results on the eigenconfiguration of the operator $Q_n = B_{n,3}$. More precisely, we investigate the monotonic properties of the eigenvalues, derive the explicit formulas for the eigenpolynomials, and analyse their limiting behaviour.

This talk is based on a joint work with Sofiya Ostrovska and Mehmet Turan from Atilim University, Türkiye.

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